

Please note: these are not full worked solutions. They are abbreviated: some parts give only the final answer, and the rest show only the key steps.

Paper 1

1.

(a) $6a - 10a + 15 = \boxed{15 - 4a}$

(b) $c(d-1) + (d-1) = \boxed{(c+1)(d-1)}$

2.

(a) $120 = 2 \times 2 \times 2 \times 3 \times 5 = \boxed{2^3 \times 3 \times 5}$

(b) (i) The HCF takes the lowest power of each prime factor:

p^1, q^1, r^3 . HCF = $\boxed{pqr^3}$

(ii) $pqr^3 = 120 = 2^3 \times 3 \times 5$.

p, q, r are primes, so $r^3 = 2^3$, giving $r = 2$, and $pq = 15$ with $p > q$, giving $p = 5, q = 3$ (and $5 > 3 > 2$ as required).

$A = pq^2r^4 = 5 \times 3^2 \times 2^4 = 5 \times 9 \times 16 = \boxed{720}$

(iii) The LCM takes the highest power of each prime factor:

$\text{LCM} = p^2q^2r^4 = 5^2 \times 3^2 \times 2^4 = \boxed{2^4 \times 3^2 \times 5^2}$

3.

(a) $OA = OC$ (radii of circle), so $\angle OCA = \angle OAC = 16^\circ$ (base $\angle$ s of isos. $\triangle OAC$).

$\angle AOC = 180^\circ - 16^\circ - 16^\circ = 148^\circ$ ($\angle$ sum of $\triangle OAC$)

$\angle BOC = \angle AOC - \angle AOB = 148^\circ - 44^\circ = \boxed{104^\circ}$

(b) $\angle BAC = \frac{1}{2}\angle BOC = \boxed{52^\circ}$

($\angle$ at centre = $2 \angle$ at circumference)

4. Numerator = $7(3x + 2y)$; denominator = $2(3x + 2y)$.

Since $x, y > 0$, $3x + 2y \neq 0$, so

$$\frac{7(3x + 2y)}{2(3x + 2y)} = \boxed{\frac{7}{2}},$$

which does not depend on x or y .

5. $\boxed{5 \text{ and } 9}$

6.

(a) (i) Time = $\boxed{\frac{5x}{36}}$ hours

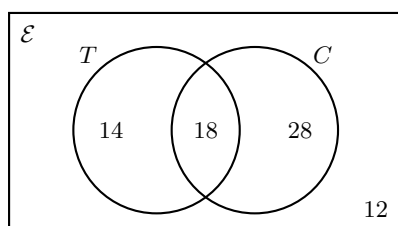
(ii) Average speed = $\frac{\text{total distance}}{\text{total time}}$
 $= \frac{72x}{5x} = \boxed{14.4}$ km/h

(b) $12 \text{ km/h} = \frac{12 \times 1000 \text{ m}}{3600 \text{ s}} = \boxed{3\frac{1}{3}}$ m/s ($\approx 3.33 \text{ m/s}$)

7. $\sin^{-1} 0.9867 = 80.645^\circ$. Since x° is obtuse,
 $x = 180 - 80.645 = 99.355 = \boxed{99.4}$ (1 d.p.)

8.

(a) $x = \boxed{18}$



(b) $(T \cap C) \cup (T \cup C)'$ is "both" together with "neither":

$18 + 12 = \boxed{30}$

(c) $P = \boxed{\frac{9}{16}}$

9.

(a) The first non-zero digit is in the 5th decimal place:

$0.0000123 = \boxed{1.23 \times 10^{-5}}$

(b) $\frac{5B}{A} = 16 \times 10 = 160 = \boxed{1.6 \times 10^2}$

10.

(a) Mid-values: 15, 22.5, 27.5, 35, 55.

$\sum fx = 15 + 45 + 220 + 245 + 385 = 910$

Estimated mean = $\frac{910}{25} = \boxed{36.4}$ minutes

(b) $\boxed{\text{Decrease}}$. Her time of 39 minutes goes into the class $30 < t \leq 40$, so the estimate counts it at that class's mid-value, 35 minutes, which is less than the estimated mean of 36.4 minutes. Adding a value below the mean lowers the mean.

11. Brian receives $\boxed{\$360.00}$

12. Readings (ms):

20-year-olds: median 384, $Q_1 = 340$, $Q_3 = 412$, IQR = $412 - 340 = 72$.

50-year-olds: median 412, $Q_1 = 360$, $Q_3 = 444$, IQR = $444 - 360 = 84$.

1. The median reaction time of the 20-year-olds (384 ms) is lower than that of the 50-year-olds (412 ms), so on average the 20-year-olds react faster.

2. The interquartile range of the 20-year-olds (72 ms) is smaller than that of the 50-year-olds (84 ms), so the 20-year-olds' reaction times are more consistent (less spread out).

13. The integers are

$\boxed{-10, -9, -8, -7, -6, -5}$

14.

(a) $\vec{OH} = \vec{OG} + \vec{GH} = \begin{pmatrix} 5 \\ -14 \end{pmatrix} + \begin{pmatrix} -8 \\ 20 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

$H = \boxed{(-3, 6)}$

(b) $\boxed{y = -\frac{5}{2}x - \frac{3}{2}}$

(c) $FG = \sqrt{(5 - (-3))^2 + (-14 - (-4))^2} = \sqrt{64 + 100}$
 $= \sqrt{164} = 12.806 = \boxed{12.8}$ units

15. $\boxed{x = -2, y = -1\frac{1}{2}}$

16.

$\frac{3c + 8c - 12}{(2c - 3)^2} = \boxed{\frac{11c - 12}{(2c - 3)^2}}$

17. $\$1 = \mathcal{L} \boxed{0.624}$

18. Let M be the point where AB touches the smaller circle, radius r ; let the larger radius be R .

$OM \perp AB$ (tan. $\perp$ rad.), so M is the midpoint of chord AB ($\perp$ from centre bisects chord): $AM = 9.5$ cm.

In $\triangle OAM$: $R^2 = r^2 + 9.5^2$ (Pythagoras' Thm), so $R^2 - r^2 = 90.25$.

Shaded area = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 90.25\pi$
 $= 283.53 = \boxed{284}$ cm²

19.

- (a) $1 - 0.37 = \boxed{0.63}$
 (b) Two red and one not red, in any of 3 orders:

$$3 \times \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18} = \frac{2700}{6840} = \boxed{\frac{15}{38}}$$

20.

- (a) Bearing of L from $B = 304^\circ - 180^\circ = \boxed{124^\circ}$
 (b) At L , the angle between North and LB is $360^\circ - 304^\circ = 56^\circ$.
 $\angle PBL = 56^\circ$ (alt. $\angle$ s, North lines parallel)
 $\angle BPL = 90^\circ$ (B due north of P , L due east of P).
 $\tan 56^\circ = \frac{PL}{18}$
 $PL = 18 \tan 56^\circ = 26.686 = \boxed{26.7}$ km
 (c) $\angle CPB = 360^\circ - 331^\circ = 29^\circ$; $\angle CBP = 90^\circ$ (C due west of B).
 $\cos 29^\circ = \frac{18}{PC}$
 $PC = \frac{18}{\cos 29^\circ} = 20.580 = \boxed{20.6}$ km

21. $\boxed{\frac{4x-5}{2x-5}}$

22. $x = -\frac{1}{4}$ (this does not make $3x - 1 = 0$)

23. $\boxed{\frac{1}{81}x^{-8}y^4z^{12}}$

(a) $a = \frac{1}{81}$, $p = -8$, $q = 4$, $r = 12$).

24.

- (a) $\angle AEB = \angle DEC$ (vert. opp. $\angle$ s)
 AB and CD are both vertical, so $AB \parallel CD$.
 $\angle BAE = \angle CDE$ (alt. $\angle$ s, $AB \parallel CD$)
 Hence $\triangle ABE$ and $\triangle DCE$ are similar (AA).
 (b) $\frac{AE}{DE} = \frac{AB}{DC} = \frac{21}{15} = \frac{7}{5}$ (corr. sides of similar $\triangle$ s)
 So $\frac{DE}{DA} = \frac{5}{12}$.
 $EF \parallel AB$ (both vertical), so $\triangle DEF$ and $\triangle DAB$ are similar
 ($\angle D$ common, $\angle DFE = \angle DBA = 90^\circ$; AA).

$$\frac{EF}{AB} = \frac{DE}{DA} = \frac{5}{12} \Rightarrow EF = \frac{5}{12} \times 21 = \boxed{8.75} \text{ m}$$

25.

(a) $\frac{216}{360} \times 2\pi r = 18\pi \Rightarrow 1.2r = 18 \Rightarrow r = \boxed{15}$ cm

(b) $V = \boxed{1020} \text{ cm}^3$

26. Multiply every number by 2, then add 15.

Multiplying by 2 doubles the mean and the standard deviation (mean 60, s.d. 20); adding 15 raises the mean by 15 and leaves the standard deviation unchanged (mean 75, s.d. 20).

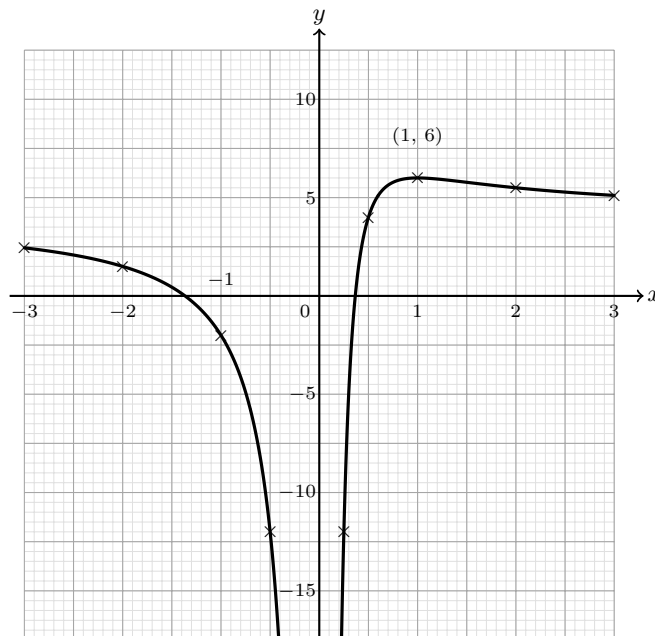
27. Percentage increase = $\boxed{300}\%$

28. $b = -6$, $c = -2$

Paper 2

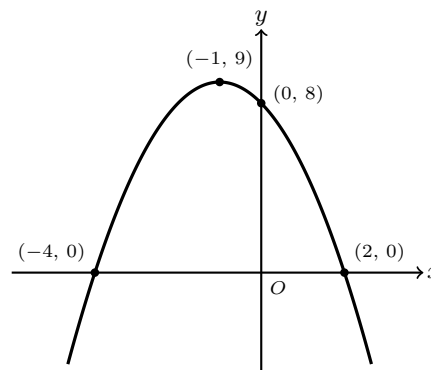
1.

- (a) (i) $x = -3$: $y = \frac{22}{9} = 2.4444$ $y = \boxed{2.4}$
 (ii) Plot the nine points and join each set with a smooth curve. The curve is not defined at $x = 0$, so it has two separate branches, which are never joined. Each branch carries on steeply downwards towards the y -axis, past $(-0.5, -12)$ and $(0.25, -12)$, to the bottom of the grid.



- (iii) The turning point (the highest point of the right-hand branch) is $\boxed{(1, 6)}$.

- (b) $y = 0$: $x = 2$ or $x = -4$. $x = 0$: $y = 8$.
 The curve is $\cap$ -shaped, with its maximum at $x = \frac{-4+2}{2} = -1$, $y = 9$.



2.

(a) $a = 3$, $b = 196$

(b) Smallest integer $n = \boxed{92}$

(c) (i) $\frac{3.99^3 + \sqrt{95.2}}{0.517} \approx \frac{4^3 + \sqrt{100}}{0.5} = \frac{64 + 10}{0.5} = \boxed{148}$

- (ii) $\boxed{\text{Overestimate}}$. 3.99 and 95.2 were both rounded up (to 4 and 100), so the numerator is too large; 0.517 was rounded down (to 0.5), and dividing by a smaller number gives a larger result. Both changes make the estimate too large.

(d) (i) n th term of $S = \boxed{6n - 8}$

- (ii) $6n - 8 = 6n - 9 + 1 = 3(2n - 3) + 1$.
 $2n - 3$ is an integer, so $3(2n - 3)$ is a multiple of 3, and every term is one greater than a multiple of 3.

(iii) (a) 6th term of $T = \boxed{17408}$

(b) Term = $\boxed{17410}$

3.

(a) $10x < 25 \Rightarrow \boxed{x < 2.5}$

(b) $4x^0 + 48x^{-2} = \boxed{4 + 48x^{-2}}$

(c) $\boxed{x = 1.79 \text{ or } x = -0.19}$

(d) (i) $\boxed{t = 4y + 3}$

(ii) $\boxed{y = \frac{7 - 4x^2}{4x^2 - 8}}$

4.

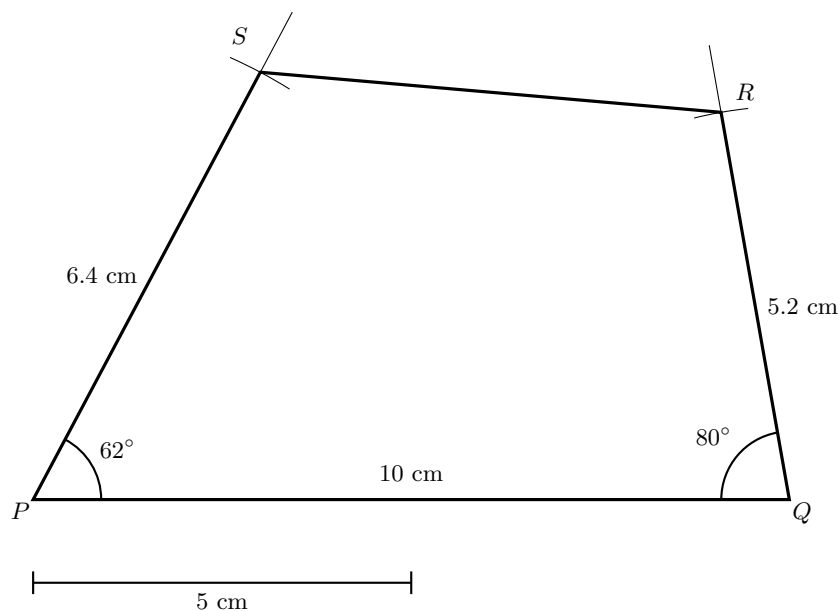
- (a) (i) Volume = $\boxed{105} \text{ m}^3$
(ii) Time = $\boxed{31.5}$ minutes
- (b) (i) Circumference of $Q = 0.8 \times$ circumference of R ;
circumference of $P = 1.1 \times 0.8 = 0.88 \times$ circumference of R .
Radius is proportional to circumference, so $r_P =$

$0.88 r_R$.

Area of $P = 0.88^2 \times$ area of $R = 0.7744 \times$ area of R .
 P is $(1 - 0.7744) \times 100\% = 22.56\%$ smaller:

$$\boxed{d = 22.56}$$

- (ii) (a) Same height, so volume $\propto r^2$: $\boxed{0.7744 : 1}$
(b) Curved surface area $= 2\pi r h$, same h , so it is $\propto r$:
 $\boxed{0.88 : 1}$



5. (a) Drawn full size.

- At P , measure $\angle QPS = 62^\circ$ with a protractor and draw the ray.
 - With compasses set to 6.4 cm and centred at P , cut the ray at S .
 - At Q , measure $\angle PQR = 80^\circ$ (inside the quadrilateral) and draw the ray.
 - With compasses set to 5.2 cm and centred at Q , cut the ray at R .
 - Join SR .
- Check: $SR \approx 6.1 \text{ cm}$, $\angle PSR \approx 113^\circ$, $\angle QRS \approx 105^\circ$.

- (b) (i) $\angle ABC = \angle AED$ (given)
 $\angle ACB = \angle CAD$ (alt. $\angle$ s, $AD \parallel BC$)
 $\angle CAD = \angle ADE$ (alt. $\angle$ s, $AC \parallel ED$)
So $\angle ACB = \angle ADE$.
 $AB = AE$ (given)
Hence $\triangle ABC \equiv \triangle AED$ (AAS).
- (ii) $AC = AD$ (corr. sides of congruent $\triangle$ s)
 $\angle ACD = \angle ADC$ (base $\angle$ s of isos. $\triangle ACD$)
 $\angle BAC = \angle EAD = y^\circ$ (corr. $\angle$ s of congruent $\triangle$ s)
 $\angle ACB = 180^\circ - x^\circ - y^\circ$ ($\angle$ sum of $\triangle ABC$)
 $\angle CAD = \angle ACB = (180 - x - y)^\circ$
(alt. $\angle$ s, $AD \parallel BC$)
In $\triangle ACD$ ($\angle$ sum of $\triangle ACD$):

$$\angle ACD = \frac{1}{2}[180 - (180 - x - y)]^\circ = \left(\frac{x + y}{2}\right)^\circ$$

(b) $(8x + 7)(2x - 3) = 0$

$$\boxed{x = 1.5 \text{ or } x = -\frac{7}{8}}$$

- (c) $x = -\frac{7}{8}$ is not valid: it gives $AC = 8(-\frac{7}{8}) - 9 = -16 \text{ cm}$
(and $AB = -\frac{3}{4} \text{ cm}$), but a length cannot be negative.

(d) $x = 1.5$: $AB = 4 \text{ cm}$, $AC = 3 \text{ cm}$.

$$\begin{aligned} BC^2 &= 4^2 + 3^2 - 2(4)(3) \cos 30^\circ \\ &= 25 - 24 \cos 30^\circ = 4.2154 \\ BC &= 2.0531 \text{ cm} \end{aligned}$$

$$\frac{\sin \angle ACB}{4} = \frac{\sin 30^\circ}{2.0531} \Rightarrow \sin \angle ACB = 0.97412$$

$$\sin^{-1} 0.97412 = 76.936^\circ; \angle ACB \text{ is obtuse, so } \angle ACB = 180^\circ - 76.936^\circ = 103.064^\circ = \boxed{103.1^\circ}$$

(Check by the cosine rule: $\cos \angle ACB = \frac{9 + 4.2154 - 16}{2(3)(2.0531)} = -0.22604$, giving 103.1° .)

6.

- (a) (i) Children = $\boxed{276}$
(ii) Mean age = $\boxed{12.8}$ years
- (b) (i) $\mathbf{T} = \begin{pmatrix} 368.40 \\ 693.55 \end{pmatrix}$
(ii) $\mathbf{D} - \mathbf{C}$ gives the profit on each cake, so the elements of $\mathbf{T}$ are Robert's profits on Day 1 and on Day 2. Their sum, \$1061.95, is the **total profit** Robert made from selling cakes over the two days.
- (c) (i) $\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$
(ii) $P(\text{at least one red and at least one blue}) = \frac{195}{256}$

7.

- (a) Area $= \frac{1}{2}(2x + 1)(8x - 9) \sin 30^\circ = \frac{1}{4}(2x + 1)(8x - 9)$.

$$\begin{aligned} \frac{1}{4}(2x + 1)(8x - 9) &= 3 \\ (2x + 1)(8x - 9) &= 12 \\ 16x^2 - 18x + 8x - 9 &= 12 \\ 16x^2 - 10x - 21 &= 0 \quad (\text{shown}) \end{aligned}$$

8.

(a) (i) $\vec{PQ} = \vec{OQ} - \vec{OP} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

(ii) $|\vec{PR}| = \boxed{2.50}$ units

(b) (i) $\vec{AB} = \vec{OB} - \vec{OA} = \boxed{-2\mathbf{a} + \mathbf{b}}$

(ii) $\vec{OC} = \frac{1}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$
 $\vec{OM} = \frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$; $\vec{ON} = \frac{7}{9}\mathbf{a} + \frac{14}{9}\mathbf{b}$

$$\begin{aligned} \vec{MN} &= \vec{ON} - \vec{OM} = \frac{1}{18}(-13\mathbf{a} + 19\mathbf{b}) \\ \vec{NC} &= \vec{OC} - \vec{ON} = \frac{2}{45}(-13\mathbf{a} + 19\mathbf{b}) \end{aligned}$$

So $\vec{NC} = \frac{4}{5}\vec{MN}$.

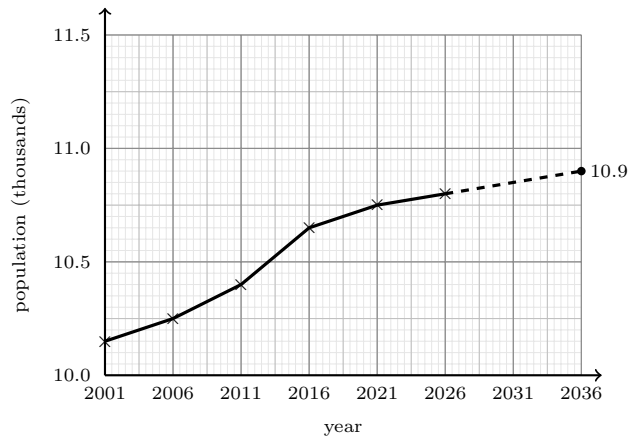
$\vec{NC}$ is parallel to $\vec{MN}$ and they share the point N , so M , N and C are collinear: MNC is a straight line.

(iii) $MN : NC = 1 : \frac{4}{5} = \boxed{5 : 4}$

9.

(a) $\frac{1}{50} \times \$160\,000 = \boxed{\$32\,000.00}$

(b)



Population in 2036 $\approx \boxed{10\,900}$

(c) Interest = \$162 045.23 (to the nearest cent).

Since \$162 045.23 < \$1 000 000, the claim is correct.

The decision does not depend on the exact estimate in (b): the interest would exceed \$1 million only if the loan were more than about \$16.3 million.